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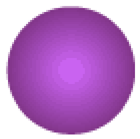
Chapter 2 : The Duality

The particle aspect of waves and The wave aspect of particles.

1) Particle Aspect of Radiation

Among the rigid concepts of classical physics:



- A particle is characterized by an energy E and a momentum $\vec{p}$,
- A wave is characterized by an amplitude and a wave vector $\vec{k}$ $\left(|\vec{k}| = {}^{2\pi}/_{\lambda}\right)$ that specifies the direction of propagation of the wave.

Particles and waves exhibit entirely different behaviors; for instance, the "particle" and "wave" properties are mutually exclusive. We should note that waves can exchange *any* (continuous) amount of energy with particles.

In this Chapter we are going to see how these rigid concepts of classical physics led to its failure in explaining a number of microscopic phenomena such as blackbody radiation, the photoelectric effect, and the Compton effect.

1.1) Blackbody Radiation

At issue here is, how radiation interacts with matter ?

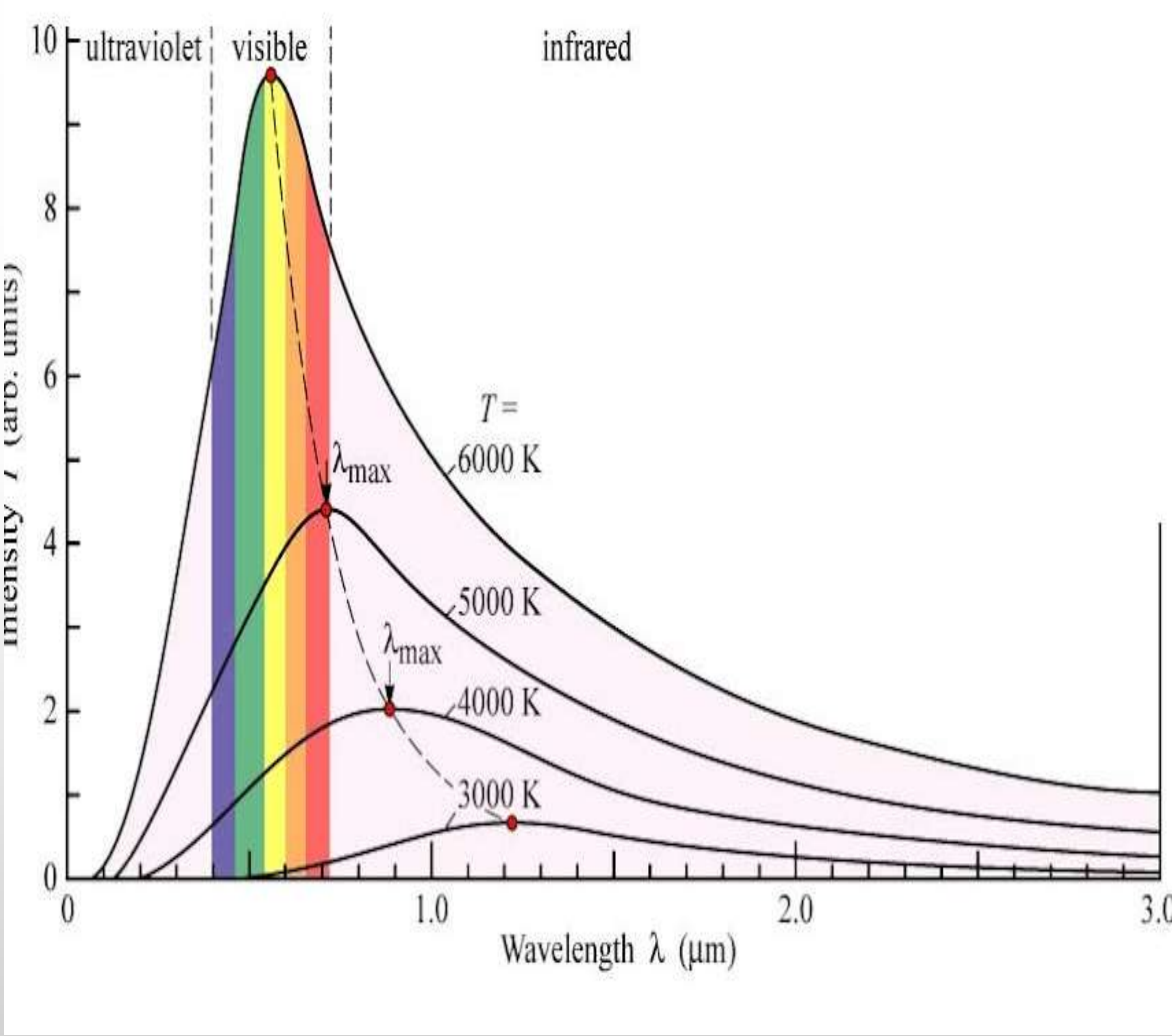
- When heated, a solid object glows and emits thermal radiation. As the temperature increases, the object becomes red, then yellow, then white. The thermal radiation emitted by glowing solid objects consists of a *continuous* distribution of frequencies ranging from infrared to ultraviolet.
- Understanding the continuous character of the radiation emitted by a glowing solid object constituted one of the major unsolved problems during the second half of the nineteenth century.
- All attempts to explain this phenomenon by means of the available theories of classical physics (statistical thermodynamics and classical electromagnetic theory) ended up in miserable failure.

1.1) Blackbody Radiation

- A *blackbody* is an object that is a perfect absorber of radiation.
- In the ideal case, it absorbs all of the light that falls on it, no light is reflected by it, and no light passes through it.
- While such an object doesn't reflect any light, if we heat up a blackbody, it can *radiate* electromagnetic energy as a result of the thermal agitation of the electrons in its surface. The intensity of this radiation depends on its frequency and on the temperature; the light it emits ranges over the entire spectrum.

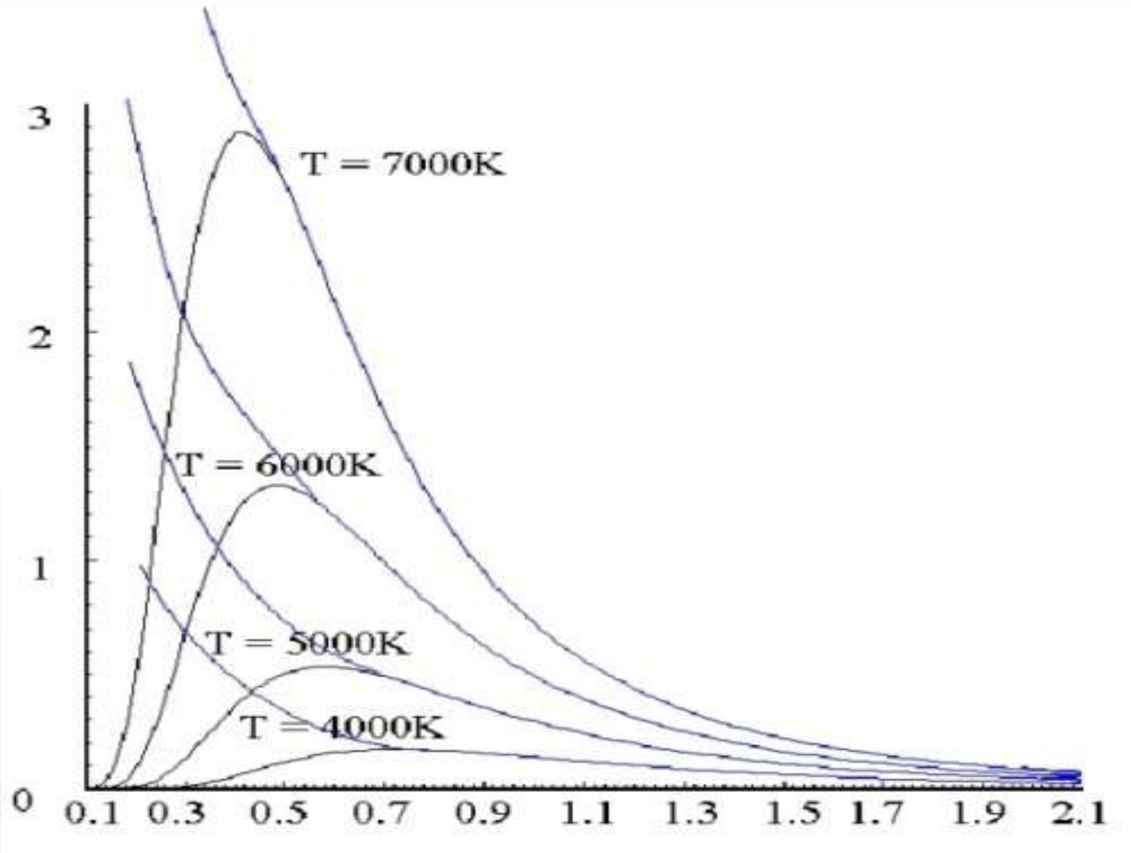
1.1) Blackbody Radiation

- ➢The total energy is all the greater as the temperature is high.
- ➢The maximum of the curve, moves towards the short wavelengths when the temperature increases.



1.1) Blackbody Radiation

- The classic theory predicts that the intensity of the radiation must continue to climb while the experiment proves the contrary.



- Why does it work for long wavelengths?
- Would there be less vibration of atoms at high frequencies?
- It is possible, but why?

1.1) Blackbody Radiation

- Planck succeeded in 1900 in avoiding the ultraviolet catastrophe and proposed an accurate description of blackbody radiation.
- He considered that the energy exchange between radiation and matter must be *discrete*.
- Planck *postulated* that the energy of the radiation (of frequency ν) emitted by the oscillating charges (from the walls of the cavity) must come *only* in *integer multiples* of $h\,\nu$:

$$\boldsymbol{E = n\,h\nu \qquad , n = 1, 2, 3, \ldots ,}$$

The constant $h = 6{,}62.\,10^{-34}$ (Joules-seconds) is called *Planck's constant*. It is frequently convenient to use the symbol $\hbar = h/2\pi$.

1.2) Photoelectric Effect

- The photoelectric effect provides a direct confirmation for the energy quantization of light

- In 1887 Hertz discovered the photoelectric effect: *electrons were observed to be ejected from metals when irradiated with light*.

1.2) Photoelectric Effect

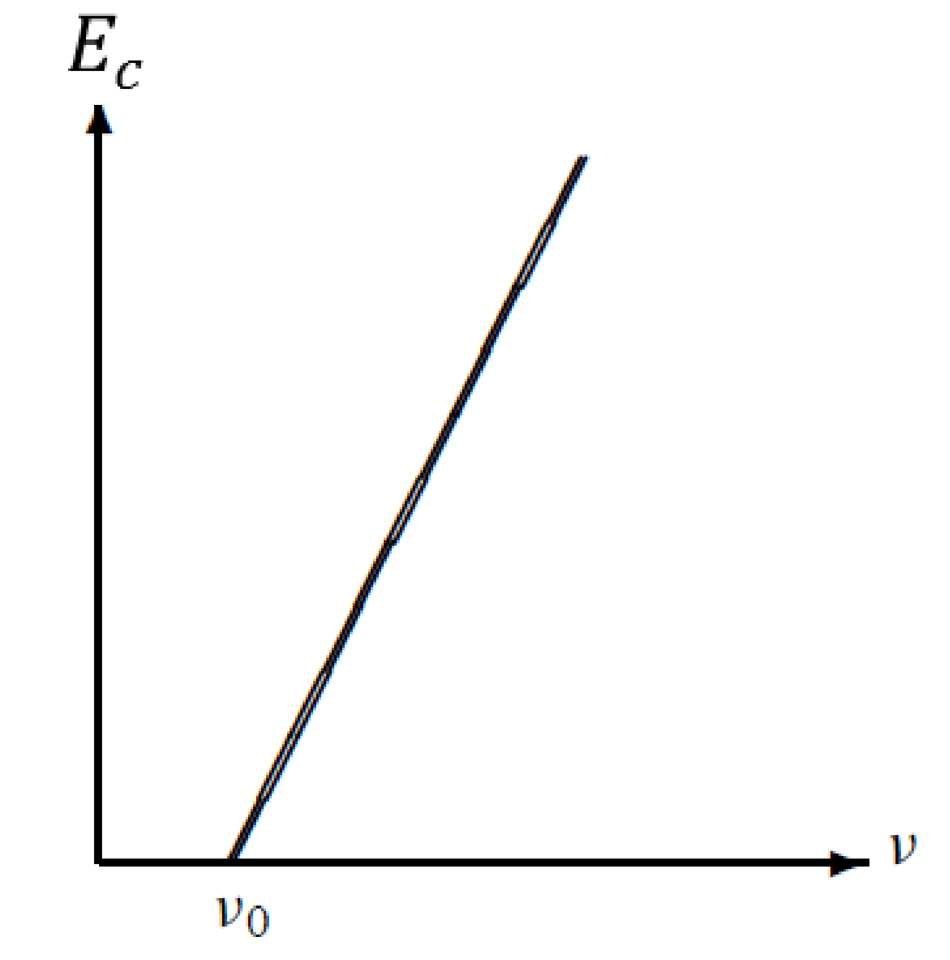
The following experimental laws were discovered prior to 1905:

- If the frequency of the incident radiation is smaller than the metal's *threshold frequency* – a frequency that depends on the properties of the metal – no electron can be emitted regardless of the radiation's intensity.

- No matter how low the intensity of the incident radiation, electrons will be ejected *instantly* the moment the frequency of the radiation exceeds the threshold frequency $\boldsymbol{\nu_0}$.

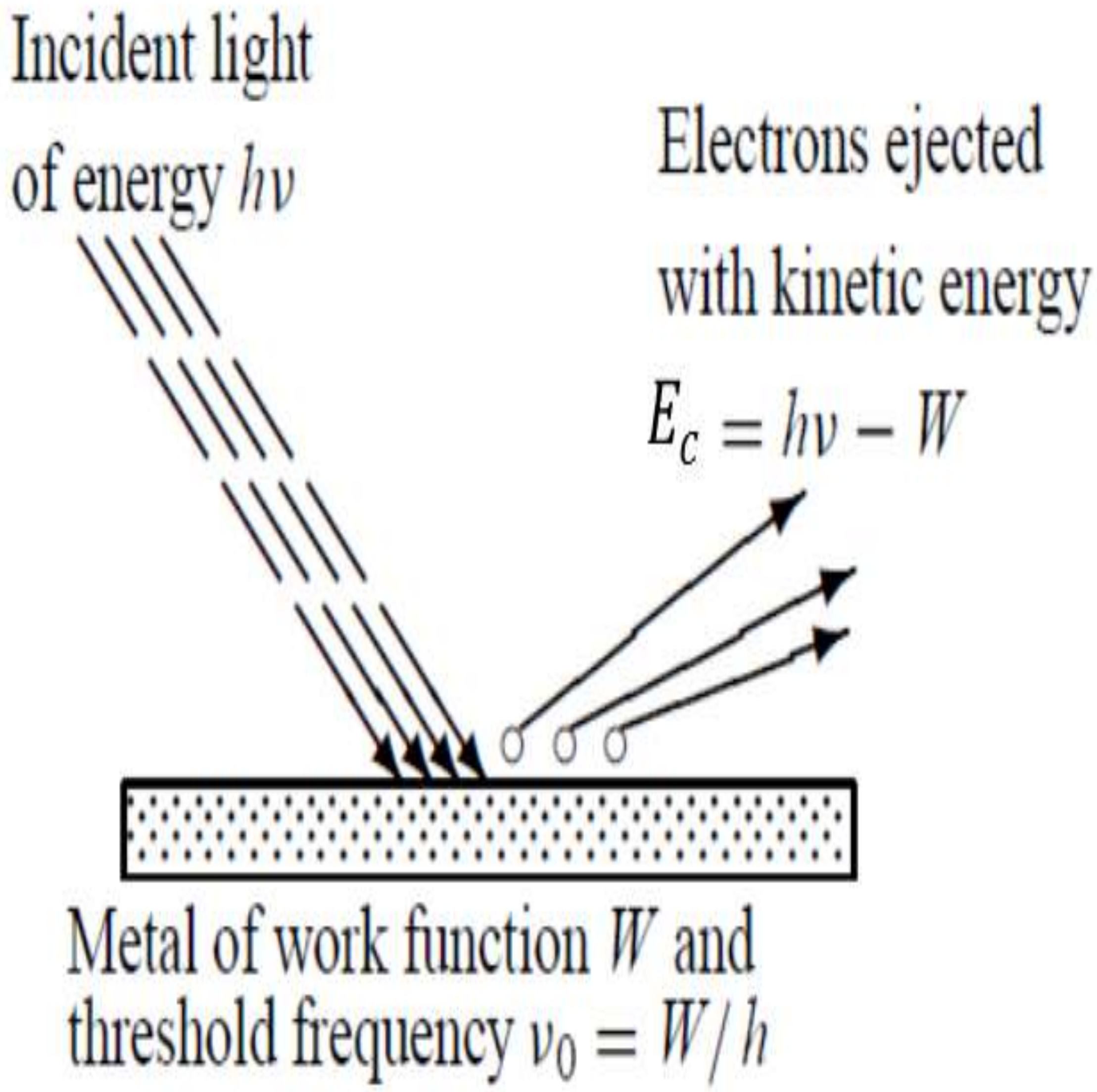
- At any frequency above $\boldsymbol{\nu_0}$, the number of electrons ejected increases with the intensity of the light but does not depend on the light's frequency.

- The kinetic energy of the ejected electrons depends on the frequency but not on the intensity of the beam; the kinetic energy of the ejected electron increases *linearly* with the incident frequency.



Kinetic energy E_c of the electron leaving the metal when irradiated with a light of Frequency $\boldsymbol{\nu}$; when $\boldsymbol{\nu < \nu_0}$ no electron is ejected from the metal regardless of the intensity of the radiation.

- When a beam of light of frequency ν is incident on a metal, each photon transmits all its energy $h\nu$ to an electron near the surface; in the process, the photon is entirely absorbed by the electron.
- The electron will thus absorb energy *only* in quanta of energy $h\nu$, irrespective of the intensity of the incident radiation.
- If $h\nu$ is larger than the metal's *work function* the electron will then be knocked out of the metal.



Activity 01

When two ultraviolet beams of wavelengths $\lambda_1 = 80\ nm$ and $\lambda_2 = 110\ nm$ fall on a lead surface, they produce photoelectrons with maximum energies 11,390 eV and 7,154 eV, respectively.

(a) Estimate the numerical value of the Planck constant.

(b) Calculate the work function, the cutoff frequency, and the cutoff wavelength of lead.

Activity 02

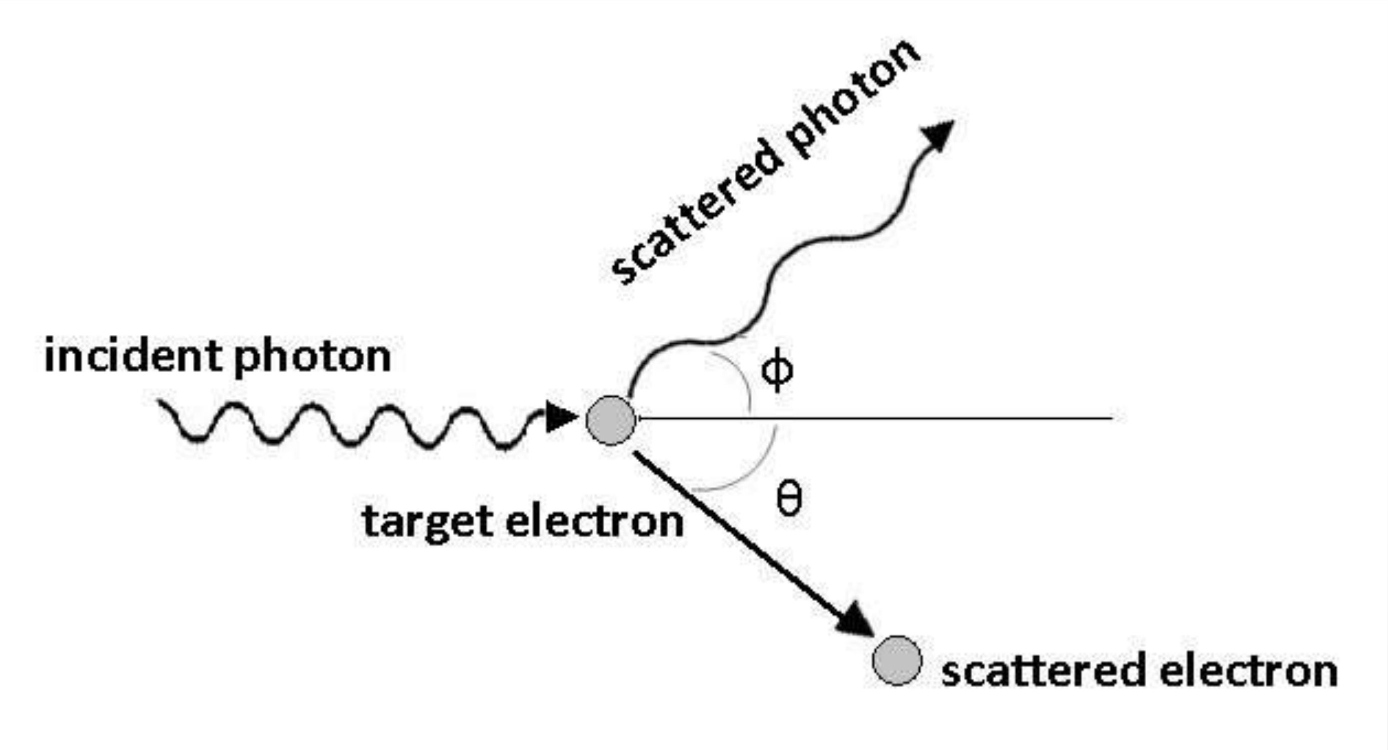
When light of a given wavelength is incident on a metallic surface, the stopping potential for the photoelectrons is 3,2 V. If a second light source whose wavelength is double that of the first is used, the stopping potential drops to 0,8 V. From these data, calculate

(a) the wavelength of the first radiation and

(b) the work function and the cutoff frequency of the metal.

1.3) Compton Effect

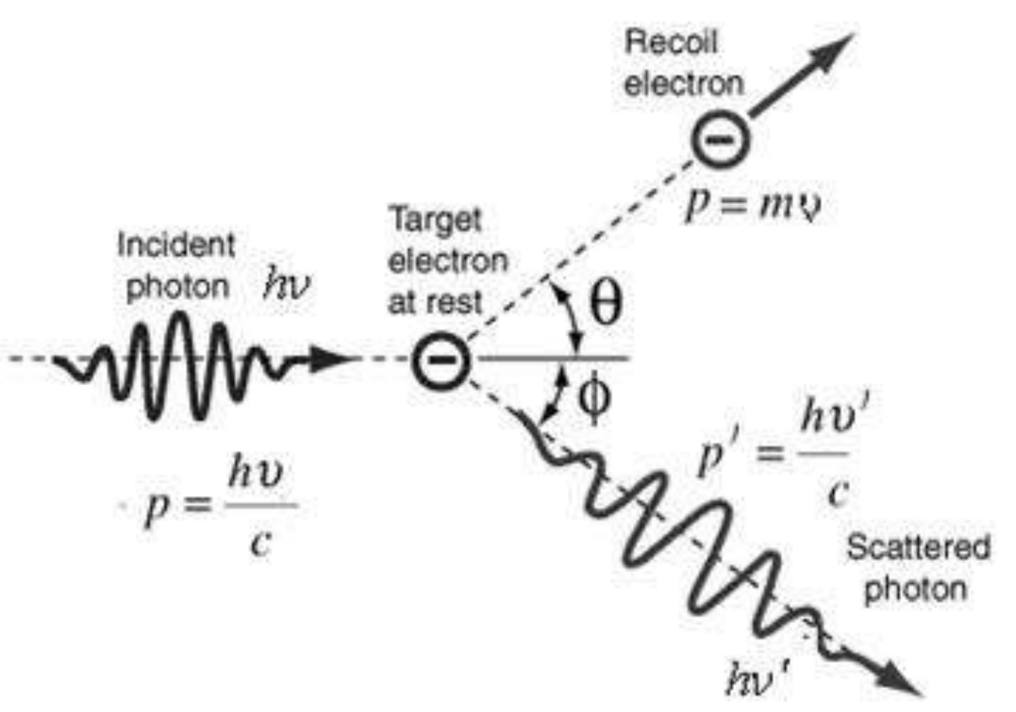
According to classical physics, the incident and scattered radiation should have the same wavelength.



But the experimental findings of Compton reveal that the wavelength of the scattered X-radiation *increases* by an amount $\Delta\lambda$.

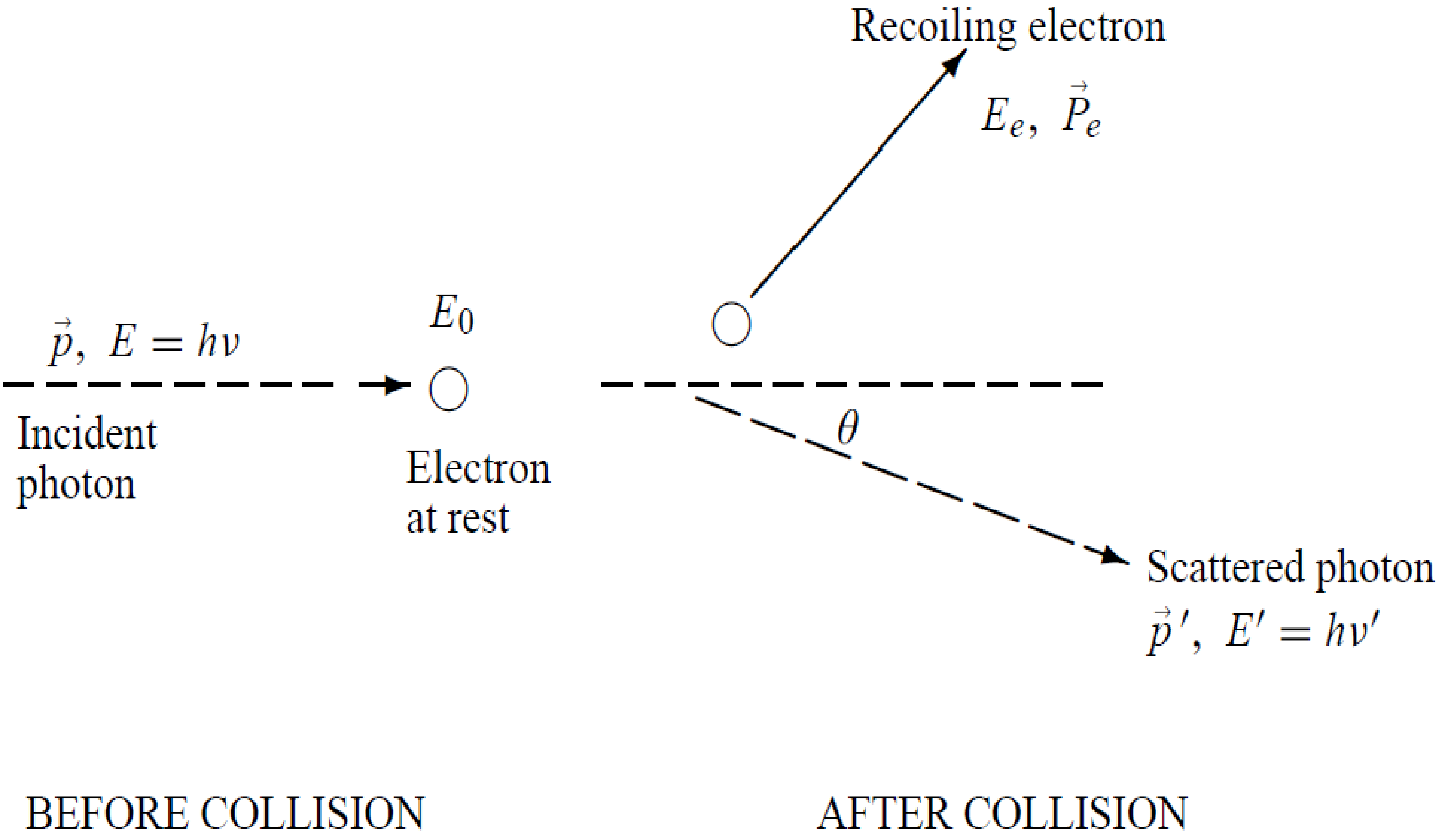
1.3) Compton Effect

- In his 1923 experiment, Compton provided the most conclusive confirmation of the particle aspect of radiation.
- By scattering X-rays off free electrons, he found that the wavelength of the scattered radiation is larger than the wavelength of the incident radiation.
- This can be explained only by assuming that the X-ray photons behave like particles.



Compton succeeded in explaining his experimental results only after treating the incident radiation as a stream of particles —photons— colliding *elastically* with individual electrons. In this scattering process, which can be illustrated by the elastic scattering of a photon from a free electron, the laws of elastic collisions can be invoked, notably the *conservation* of energy and momentum.

$$\begin{cases} Momentum\ Before\ Collision = Momentum\ After\ Collision \\ Energy\ Before\ Collision = Energy\ After\ Collision \end{cases}$$


Recoiling electron
$E_e,\ \vec{P}_e$
E_0
$\vec{p},\ E = h\nu$
Incident photon
Electron at rest
θ
Scattered photon
$\vec{p}\,',\ E' = h\nu'$
BEFORE COLLISION
AFTER COLLISION

The conservation of momentum gives :

$$\vec{p} = \overrightarrow{p_e} + \overrightarrow{p'}$$

$$\overrightarrow{p_e} = \vec{p} - \overrightarrow{p'}$$

$$(\overrightarrow{p_e})^2 = \left(\vec{p} - \overrightarrow{p'}\right)^2$$

$$p_e^2 = p^2 + p'^2 - 2pp'cos\theta \quad \text{with } p = \frac{h\nu}{c}$$

$$p_e^2 = \frac{h^2}{c^2}(\nu^2 + \nu'^2 - 2\nu\nu'cos\theta)$$

Let us now turn to the energy conservation

$$E + E_0 = E_e + E'$$

The energies of the electron before and after the collision are given, respectively, by:

$$E = h\nu$$

$$E_0 = m_0 c^2$$

$$E_e = \sqrt{p_e^2 c^2 + m_0^2 c^4}$$

$$E_e = \sqrt{\frac{h^2 c^2}{c^2}(\nu^2 + \nu'^2 - 2\nu\nu' cos\theta) + m_0^2 c^4}$$

$$E_e = h\sqrt{(\nu^2 + \nu'^2 - 2\nu\nu' cos\theta) + \frac{m_0^2 c^4}{h^2}}$$

$$E' = h\nu'$$

$$h\nu + m_0 c^2 = h\nu' + h\sqrt{(\nu^2 + \nu'^2 - 2\nu\nu' cos\theta) + \frac{m_0^2 c^4}{h^2}}$$

which in turn leads to: $\frac{h\nu - h\nu' + m_0c^2}{h} = \sqrt{(\nu^2 + \nu'^2 - 2\nu\nu' cos\theta) + \frac{m_0^2c^4}{h^2}}$

$$(\nu - \nu') + \frac{m_0c^2}{h} = \sqrt{\left(\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta\right) + \frac{m_0^2c^4}{h^2}}$$

$$\left[(\nu - \nu') + \frac{m_0c^2}{h}\right]^2 = \left[\sqrt{\left(\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta\right) + \frac{m_0^2c^4}{h^2}}\right]^2$$

$$(\nu - \nu')^2 + \left(\frac{m_0c^2}{h}\right)^2 + 2\left(\frac{m_0c^2}{h}\right)(\nu - \nu') = \left(\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta\right) + \frac{m_0^2c^4}{h^2}$$

$$\nu^2 + \nu'^2 - 2\nu\nu' + \frac{m_0^2c^4}{h^2} + 2\left(\frac{m_0c^2}{h}\right)(\nu - \nu') = \nu^2 + \nu'^2 - 2\nu\nu'\cos\theta + \frac{m_0^2c^4}{h^2}$$

$$-2\nu\nu' + 2\left(\frac{m_0c^2}{h}\right)(\nu - \nu') = -2\nu\nu'\cos\theta$$

$$\left(\frac{m_0c^2}{h}\right)(\nu - \nu') = \nu\nu'(1 - \cos\theta) \Leftrightarrow \frac{(\nu - \nu')}{\nu\nu'} = \left(\frac{h}{m_0c^2}\right)(1 - \cos\theta)$$

$$\frac{1}{\nu'} - \frac{1}{\nu} = \frac{h}{m_0c^2}(1 - \cos\theta)$$

Hence the wavelength shift is given by

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}\left(1-\cos\theta\right) = 2\lambda_C \sin^2\left(\frac{\theta}{2}\right)$$

λ_C

where $$\lambda_C = \frac{h}{m_0 c} = 2,426.\ 10^{-12}\, m$$

is called the Compton wavelength of the electron.

Activity 03

A 5.5-MeV gamma ray is scattered al 60° from an electron.

What is the energy in megaelectronvolts (MeV) of the scattered photon?

2.1) de Broglie's Hypothesis: Matter Waves

In 1923 de Broglie suggested that :

"all material particles should also display a dual wave–particle behavior "

So, starting from the momentum of a photon $p = \frac{h\nu}{c}$, we can generalize this relation to any material particle with nonzero rest mass: *each material particle of momentum $\vec{p}$ behaves as a group of waves* (matter waves) whose wavelength λ and wave vector $\vec{k}$ are governed by the speed and mass of the particle

$$\lambda = \frac{h}{p} \leftrightarrow p = \frac{h}{\lambda} \leftrightarrow p = \frac{h}{2\pi}\frac{2\pi}{\lambda} \leftrightarrow p = \hbar\vec{k}, \quad alors \qquad \vec{k} = \frac{\vec{p}}{\hbar}$$

The *de Broglie relation*, connects the momentum of a particle with the wavelength and wave vector of the wave corresponding to this particle.

Activity 04

A thermal neutron has a speed *v* that corresponds to room temperature *T* = 300 K.

What is the wavelength of a thermal neutron?

Knowing that:

$$m_n = (1{,}67.10^{-27})\ kg$$

$$h = (6{,}23.10^{-34})\ J.s$$

$$k = (1{,}38.10^{-23})\ J.k^{-1}$$