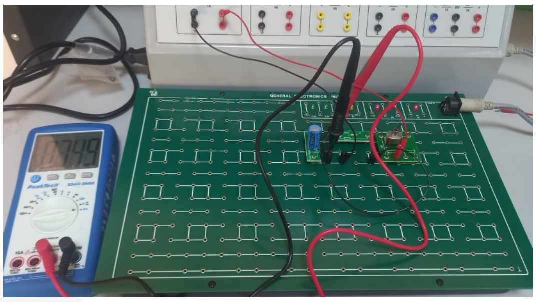
Physics II Electricity and magnetism



Dr. F. ZAMI

University of Tissemsilt

Faculty of Science and Technology

Departement of Science and Technology

Email; f.zami@univ-tissemsilt.dz

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Objectifs

Introduce to the student the ***physical phenomena*** underlying ***the laws of electricity*** in general.

Introduction

Electricity is a fundamental force of nature that plays a pivotal role in modern society, powering a vast array of devices and systems that have become essential to our daily lives. It is a form of energy resulting from the existence of ***charged particles (such as electrons or protons)***, either statically as an accumulation of charge or dynamically as a current flow. The study and harnessing of electricity have significantly shaped the course of human history and technological progress.

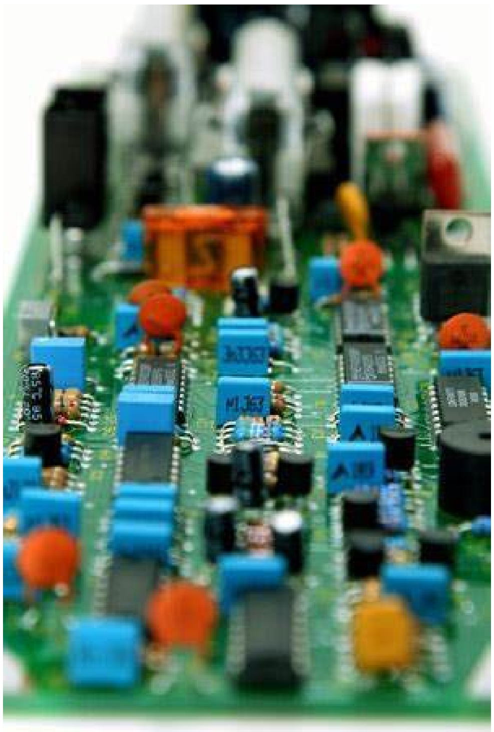
The story of electricity dates back to ancient times when Greek philosophers observed static electricity generated by rubbing certain materials together, such as amber and fur. However, it wasn't until the ***17th*** century that scientists like ***William Gilbert*** began to systematically explore electrical phenomena. The groundbreaking work of ***Benjamin Franklin*** in the ***18th*** century brought key insights, including the identification of positive and negative charges and the concept of electric currents.

The invention of the electric battery by ***Alessandro Volta*** in the early ***19th*** century marked a crucial milestone, providing a reliable source of continuous electric current. ***Michael Faraday***'s experiments with electromagnetism and induction further advanced our understanding of electricity, laying the foundation for practical applications.

The late ***19th*** and early ***20th*** centuries witnessed the development of the electric power industry, with pioneers like ***Thomas Edison*** and ***Nikola Tesla*** contributing significantly. Edison is known for his work on ***direct current (DC)*** systems, while Tesla championed ***alternating current (AC)***, which eventually became the dominant form of electric power transmission.

Today, electricity powers an extensive range of devices and systems, from ***lighting*** and ***heating*** to ***complex electronic devices*** and ***industrial machinery***. It is generated through various means, including ***fossil fuels***, ***nuclear power***, and ***renewable sources*** like ***solar*** and ***wind energy***. The global electrical grid facilitates the distribution of electricity over vast distances, connecting power generation facilities to ***homes***, ***businesses***, and ***industries.***

Understanding and harnessing the principles of electricity have not only revolutionized our daily lives but have also fueled innovation in fields such as ***telecommunications***, ***medicine***, and ***transportation***. The ongoing quest for cleaner and more sustainable energy sources continues to shape the future of electricity, driving advancements in technology and fostering a more electrified and interconnected world.



I Prior Knowledge

1. Prior Knowledge

The student must have a solid background in ***mathematics*** and ***physics***. he must know firstly the difference between ***scalar*** and ***vector*** quantities. Secondly; the ***diverse coordinates systems*** in particular ***the displacement***, ***the position***, ***the speed*** and ***the acceleration equations***. Thirdly, he must know how to deal with the different ***operators*** (***Nabla*** and ***Laplacian***). Finally, he must do well with ***multiple integrals*** and ***derivatives***.

Test of Prior Knowledge

<u>Exercise N° 01:</u> -Complete the table

	$\overrightarrow{\mathbf{OM}}$	$\overrightarrow{\mathbf{dl}}$	$\vec{\mathbf{V}}$	$\vec{\mathbf{a}}$
Cartesian $(\vec{\mathbf{i}}, \vec{\mathbf{j}}, \vec{\mathbf{k}})$				
Polar $(\vec{\mathbf{e}_\rho}, \vec{\mathbf{e}_\theta})$				
Cylindrical $(\vec{\mathbf{e}_\rho}, \vec{\mathbf{e}_\theta}, \vec{\mathbf{e}_z})$				
Spherical $(\vec{\mathbf{e}_\rho}, \vec{\mathbf{e}_\theta}, \vec{\mathbf{e}_\varphi})$				
Frenet $(\vec{\mathbf{e}_B}, \vec{\mathbf{e}_T}, \vec{\mathbf{e}_N})$				

<u>Exercise N°2 :</u>

I- Given the following vectors:

$\vec{\mathbf{r_1}} = \mathbf{3t^2\vec{i} + 2t^3\vec{j} - t\,\vec{k}}$ and $\vec{\mathbf{r_2}} = \mathbf{4t\vec{i} + t\vec{j} + t\vec{k}}$

Calculate $\frac{d}{dt}(\vec{\mathbf{r_1}}.\vec{\mathbf{r_2}})$

- By using the derivatives of vectors rules.
- By calculating the product $\vec{\mathbf{r_1}}.\vec{\mathbf{r_2}}$ then by derivating

Given the following vectors: $\vec{\mathbf{r_1}} = \mathbf{e^{-at}\vec{i} + e^{-2at}\vec{j}}$ and $\vec{\mathbf{r_2}} = \mathbf{e^{-2at}\vec{j} + e^{-at}\vec{k}}$

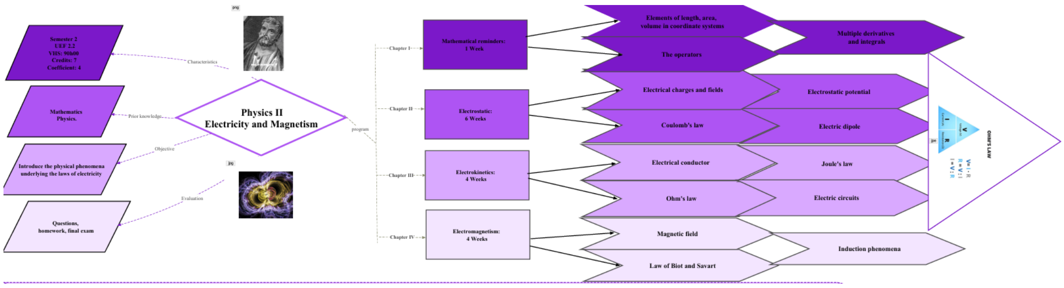
- Calculate $\frac{d}{dt}(\vec{\mathbf{r_1}} \times \vec{\mathbf{r_2}})$ By using the derivatives of vectors rules.
- By calculating the product $\vec{\mathbf{r_1}} \times \vec{\mathbf{r_2}}$ then by derivating.

<u>Exercise N°3:</u>

1) Given the scalar field $\boldsymbol{\varphi} = \mathbf{x^2 + y^2 = r^2}$, find the gradient of $\boldsymbol{\varphi}$.
2) Let $[\boldsymbol{\nabla}\mathbf{Q} = (\mathbf{1 + 2xy})\,\vec{\mathbf{i}} + (\mathbf{x^2 + 3y^2})\vec{\mathbf{j}}]$ find the associated scalar field.
3) Calculate the divergent of the position vectors field $\vec{\mathbf{r}} = \mathbf{x\vec{i} + y\vec{j} + z\vec{k}}$
4) Calculate the rotational of the vectors field $\vec{\mathbf{A}} = (\mathbf{3x^2y\vec{i} + yz^2\vec{j} - xz\vec{k}})$

II Conceptual Card

1. Conceptual Card



Conceptual card

III Chapter I: Mathematical Review

1. Objectives

1.1. Objectives

- Know the definitions of the basic concepts in electricity.

- Differentiate between the coordinates systems, operators... etc.

2. I- Elements of length, area, volume in coordinate systems, Solid angle, Operators and Multiple derivatives and integrals

2.1. I-1 Cartesian coordinate system

Définition

A Cartesian coordinate system is defined by an origin point ***O*** and three perpendicular axes

(O x, O y, O z). The unit vectors along the axes area, .Each point ***M*** in space is identified by the three components of the vector ***R*** connecting ***O*** to ***M***, (see Figure I.1):

-***The elementary displacement***: $d\,l = d\,OM = d\,x.\ i + d\,y.\ j + dz\ .k$

-***The elementary volume*** : $d\,V = d\,x.\ d\,y.\ d\,z$

-***The elementary area***: $d\,S = d\,x^2.\ d\,y^2.\ d\,z^2$

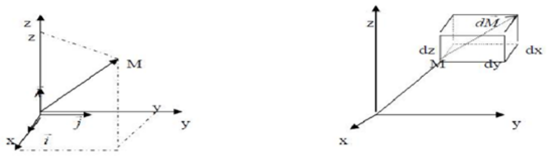


Figure I.1: Cartesian basis, Position vector, elementary displacement and volume.

2.2. I-2 Cylindrical coordinate system

The cylindrical base u_ρ, u_θ, k is obtained by rotating i, j, k by an angle θ around the ***Oz*** axis (Figure I.2).

-***The position vector*** : $OM = \rho\ u_\rho + z\ k$

-***The elementary displacement***: $d\,l = d\rho\ u_\rho + \rho\ d\,\theta\ u_\theta + dz\ k$

-***The elementary volume*** : $d\,V = \rho\ d\rho\ d\,\theta\ dz$

-***The elementary area***: $d\,S = \rho\ d\rho\ d\,\theta$

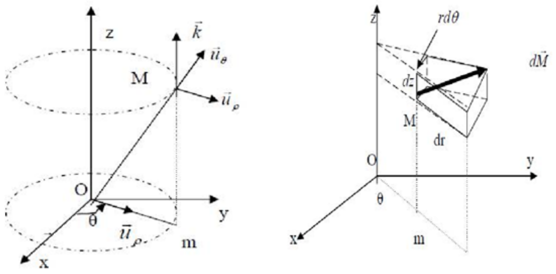


Figure I.2: Cylindrical basis, Position vector, elementary displacement and volume.

2.3. I-3 Spherical coordinate system

The three vectors $\boldsymbol{u}_{\rho}$, $\boldsymbol{u}_{\theta}$, $\boldsymbol{u}_{\varphi}$ forming ***the spherical base***, that can be obtained by a rotation of

$\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ by an angle $\boldsymbol{\varphi}$ around ***Oz***, followed by a rotation by an angle $\boldsymbol{\theta}$ around $\boldsymbol{u}_{\varphi}$.

-***The position vector*** : $OM = \rho\, u_{\rho} = x\, i + y\, j + z\, k$

-***The elementary displacement***: $d\, l = d\rho\, u_{\rho} + \rho\, d\theta\, u_{\theta} + \rho\, d\varphi\, \sin\theta\, u_{\varphi}$

-***The elementary volume*** : $d\, V = \rho^{2}\, d\rho\, \sin\theta\, d\theta\, d\varphi$

-***The elementary area***: $d\, S = \rho^{2} \sin\theta\, d\theta\, d\varphi$

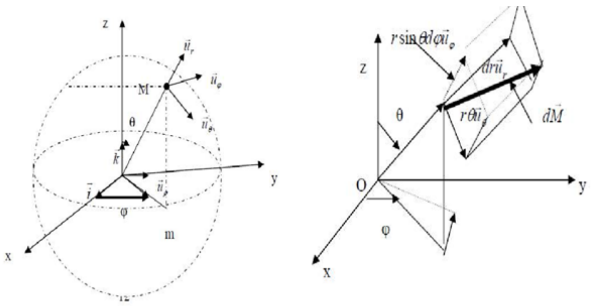


Figure I.3: Spherical basis, Position vector, elementary displacement and volume.

Exemple

1- Calculate ***the perimeter*** of a circle ***C*** of radius ***R*** (simple integral).

2- Calculate ***the area*** of a disk ***D*** of radius ***R*** (double area integral).

3- Calculate ***the volume of a cylinder V*** of radius ***R*** and height ***H*** (triple integral of volume).

4- Calculate ***the area of a half-sphere D*** of radius ***R*** (without the horizontal disk) (double integral of surface).

5- Calculate ***the volume of a sphere V*** of radius ***R*** (triple integral of volume).

Solution:

1-We have $d\,l=R\,d\,\theta$ where $C=\int R\,d\,\theta=2\,\pi\,R$

2-We have $d\,S=R\,d\,R\,d\theta$ where $D=\iint R\,d\,R\,d\theta=\int R\,d\,R\,.\int d\theta=\pi\,R^{2}$

3-We have $V=\iiint R\,d\,R\,d\theta\,dz=\int R\,d\,R\int d\theta\int dz=\pi\,R^{2}\,H$

4-We have $d\,S=R^{2}\,\sin\theta\,d\varphi\,d\theta,\ D=\iint R^{2}\,\sin\theta\,d\varphi\,d\theta=R^{2}\int\sin\theta\,d\theta\int d\varphi=2\,\pi\,R^{2}$

5-We have $V=\iiint R^{2}\,d\,R\,\sin\theta\,d\varphi\,d\theta=\int R^{2}\,d\,R\int\sin\theta\,d\,\theta\int d\varphi=R^{3}/3\ 2.2\,\pi=4/3\,\pi\,R^{3}$

2.4. I-4 Solid angle

Définition

A solid angle is a three-dimensional analog of an angle in two dimensions. It is a measure

of the amount of spatial extent subtended by a cone at a point in three-dimensional space.

The solid angle is expressed in Steradians (symbol: ***Sr.***). Mathematically, if you have a surface in three-dimensional space, and you consider all the rays emanating from a point on that surface and extending outward, the solid angle is the measure of the cone formed by those rays as they intersect the surface. It is defined as ***the ratio of the surface area of the cone's base (on the surface) to the square of the distance from the point to the base of the cone.***

The formula for solid angle Ω in terms of surface area A and distance r is:

$\Omega= A/r^{2}$ (Steradians).

In physics and engineering, solid angles are often used in discussions related to ***radiation,***

optics and electromagnetic fields. The total solid angle around a point in space is 4π

Steradians, which corresponds to the entire surface of a unit sphere centered at that point.

2.5. I-5 Operators (Gradient, Rotational, Nabla, Laplacian and Divergence)

Rappel

Gradient, Rotational, Nabla and Divergence are previously treated in the first semester

In physics I (point mechanics) in the last part of vector Calculations.

I-5-1: Laplacian :

The Laplacian is a scalar operator that represents ***the divergence of the gradient of a scalar field***. If ***f(x, y, z)*** is a scalar function, then the Laplacian is given by:

$\nabla^2 f=(d^2 f/ d x^2) +(d^2 f/ d y^2 +(d^2 f / d z^2)$

These operators play crucial roles in describing and understanding various physical phenomena, including ***fluid dynamics, electromagnetism, and heat conduction***, among others.

Exemple

-Suppose we have a scalar function $f(x, y, z)=x^2+y^2$

-Calculate the Laplacian of this function($\nabla^2 f$).

Solution:

-The function f is defined in two dimensions, so the Laplacian is given by:

$\nabla^2 f=d^2 f/ dx^2 +(d^2 f/ d y^2)$

-Let us find the partial derivatives:

$- d f/ d x=2 x , d^2 f/ d x^2 =2$

$- d f/ d y=2 y , d^2 f/ d y^2 =2$

-Now, plug these into the Laplacian formula:

$\nabla^2 f=d^2 f/ dx^2 +(d^2 f/ d y^2)=2+2=4$.

2.6. I-6 Multiple derivatives and integrals

I-6-1: Multiple Derivatives

1. ***First Derivativ***e: The first derivative of a function ***f(x)*** with respect to ***x*** is denoted As $f'(x)$ or $d f/ d x$.

2. ***Second Derivative***: The second derivative is the derivative of the first derivative. It is denoted as $f''(x)$ or $d^2 f / dx^2$

3. ***Higher-Order Derivatives***: The nth derivative, denoted $f^n (x)$ or $d^n f/ d x^n$, is obtained by taking the derivative ***n*** times.

I-6-2: Multiple Integrals

1. ***Single Integrals***: The definite integral of a function ***f(x)*** from ***a*** to ***b*** is denoted as $\int f (x) d x$

2. ***Double Integrals***: The double integral of a function ***f(x, y)*** over a region ***R*** is denoted as $\iint R\ f(x, y)\ d A$, where ***d A*** is the area element.

3. ***Triple Integrals***: The triple integral of a function ***f(x, y, z)*** over a region ***V*** is denoted as ***∭f(x, y, z) d V***, where ***d V*** is the volume element.

2.7. TD N 1

Exercise 1:

Given the following vectors:

$$\vec{\mathbf{A}} = (-3\vec{\mathbf{i}} - 6\vec{\mathbf{j}} + 2\vec{\mathbf{k}})\ ,\ \vec{\mathbf{B}} = (-\vec{\mathbf{i}} + 3\vec{\mathbf{j}} + 4\vec{\mathbf{k}}) \text{ and } \vec{\mathbf{C}} = (+\vec{\mathbf{i}} + 3\vec{\mathbf{j}} + 4\vec{\mathbf{k}})$$

- Calculate the scalar products $\vec{\mathbf{A}}.\vec{\mathbf{B}}$,$\vec{\mathbf{B}}.\vec{\mathbf{C}}$ deduce the angle formed by the vectors
- Calculate the cross products $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$, $\vec{\mathbf{B}} \times \vec{\mathbf{C}}$
- Find the volume of the parallelepiped formed by the vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$ and $\vec{\mathbf{C}}$
- Demonstrate that $\vec{\mathbf{A}}.(\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = \vec{\mathbf{C}}.(\vec{\mathbf{A}} \times \vec{\mathbf{B}}) = \vec{\mathbf{B}}.(\vec{\mathbf{C}} \times \vec{\mathbf{A}})$
- Calculate $\mathbf{Div}\ \overline{\mathbf{A}}$ and $\mathbf{Rot}\ \overline{\mathbf{A}}$

Exercise 2:

Consider three free vectors $\vec{\mathbf{A}}\ \vec{\mathbf{B}}\ \vec{\mathbf{C}}$

- Demonstate that they verify the relation :

$$\vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) + \vec{\mathbf{C}} \times (\vec{\mathbf{A}} \times \vec{\mathbf{B}}) + \vec{\mathbf{B}} \times (\vec{\mathbf{C}} \times \vec{\mathbf{A}}) = \vec{\mathbf{0}}$$

- Demonstate that if $\vec{\mathbf{A}}\ \vec{\mathbf{B}}\ \vec{\mathbf{C}}$ have the same magnitude, so the vectors $(\vec{\mathbf{A}} \times \vec{\mathbf{B}}) \times (\vec{\mathbf{A}} \times \vec{\mathbf{C}})$, $(\vec{\mathbf{B}} \times \vec{\mathbf{C}}) \times (\vec{\mathbf{B}} \times \vec{\mathbf{A}})$, $(\vec{\mathbf{C}} \times \vec{\mathbf{A}}) \times (\vec{\mathbf{C}} \times \vec{\mathbf{B}})$ have also the same magnitude.

Exercise 3:

Consider the coordinates of a point $\mathbf{M}\ (\mathbf{1}, \mathbf{1}, \sqrt{\mathbf{2}})$ in Cartesian coordinate system $(\vec{\mathbf{i}}, \vec{\mathbf{j}}, \vec{\mathbf{k}})$

1- Calculate the Cylindrical coordinates of M.

2- Calculate the Spherical coordinates of M.

3- In each question, give a graphical representation.

IV Chapter II Electrostatic

1. Objectives

1.1. Objectives

- Know the differences between the electrical field and potential.

- Understanding the electrostatic effects.

- Analyze electrical circuits.

2. II-1 Electrostatic charges and fields, Electrostatic interaction force

2.1. II-1-1 Electrostatic charges and fields

The electrical phenomena were discovered very early in human history. Initially, they were a source of curiosity or fear, and later became the subject of spectacular experiments in the ***16th*** and ***17th*** centuries. The scientific analysis of these phenomena between ***1785*** and ***1875*** led to the development of a coherent theory of electricity, which remains valid today with no essential modifications.

Approximately ***2600 years ago, Thales of Milets*** observed that yellow amber, a fossilized

resin used to make jewelry, when rubbed with a cat's fur, attracts small, light objects such as

bits of straw or feathers. It is worth noting at this point in history that yellow amber is called

"***Elektron***" in Greek, which is the origin of the word "***electricity***".

Electricity is called static when it accumulates on bodies due to friction or other processes.

Electrostatics is the branch of physics that studies the phenomena created by static electric

Charges for the observer.

Electrification is the phenomenon of the appearance of an electric charge or the accumulation of quantities of electricity on a body. There are three types of electrification:

- ***Frictional electrification***
- ***Contact electrification***
- ***Influence electrification.***

Exemple

Let us take a metal sphere and suspend it by a thread. Next, we approach a glass rod after

having previously rubbed it. We observe that ***the rod attracts the sphere.***

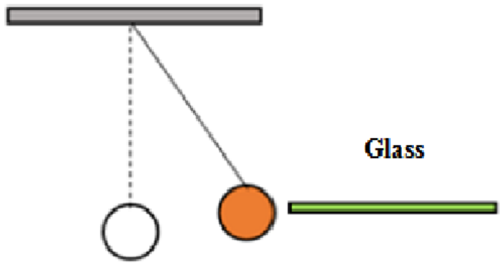


Figure II.1: Contact electrification

Exemple

Approaching a rubbed glass rod to pieces of paper with a woolen cloth, ***the latter are attracted To the rod.***

The friction on the rod caused it to lose electrons, resulting in the glass becoming positively

charged. When the rod is brought close to neutral pieces of paper, negative charges move

within the paper toward the positive charges on the glass rod. This attraction between the two objects of opposite charges is due to the presence of electrostatic forces.

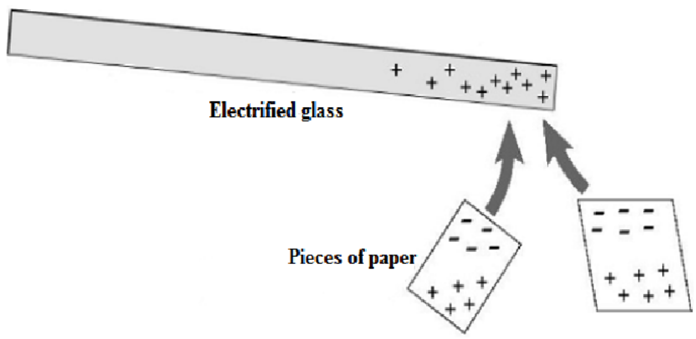


Figure II.2: Frictional electrification

2.2. II-1-2 Electric charge

Définition

It is an abstract concept comparable to that of mass, which helps explain certain behaviors.

Electric charge possesses remarkable properties that we can analyze:

1. ***Positive and negative charge***: Electric charge can exist in two forms, one called

positive and the other negative. It is an extensive quantity, meaning it can be expressed as the algebraic sum of the elementary charges that constitute it.

2. ***Conservation of electric charge***: Elementary electric charges being permanent, if a

body is isolated, meaning it cannot exchange charges with the outside, its electric charge

remains conservative and constant. This constitutes the law of conservation of charge, a

fundamental postulate of electromagnetism never experimentally contested.

3. ***Quantification of electric charge***: Numerous experiments, the most famous of which

was conducted by the American physicist ***P. Millikan in 1910***, show that the electric charge

of a system can only vary in multiples of an elementary charge with a value of $e= 1,6.10^{-19}$ ***C***, the unit C in the International System being ***the Coulomb***. The charge of any system can be written as $Q = n e$, where n is a positive or negative integer. This expression assumes that the absolute values of positive and negative elementary charges are the same. It is noteworthy that The usual stable carriers of these two types of charges have very different masses;

for the Electron: $q=-e$, $m_e=9.10^{-31} kg$

for the Proton: $q=+e$, $m_p=1,67.10^{-27}$ kg

4. Invariance of electric charge: The electric charge of a system is invariant under a

change of reference frame, meaning its value does not depend on the reference frame in which it is measured.

Exemple

Calculate the charge of ***$1,35 \times 10^{17}$ electrons***? ***(Q= - 0, 0216 C)***

2.3. II-1-3 Coulomb's law: Electrostatic force in a vacuum

Let us consider two point charges q_1 and q_2 placed in a vacuum. The first exerts on the

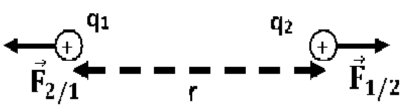
second a force proportional to its charge q_1. Conversely, the second exerts on the first a force proportional to its charge q_2 . It is deduced that the force between two point charges, called ***Electrostatic force*** is proportional to the product of their charges q_1 and q_2. It is

mathematically expressed as: $F_e=k\, q_1 q_2 /r^2 .\, U$ where: r is the distance separating the two charges;

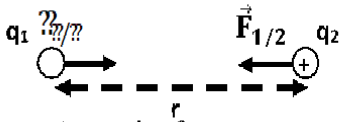
U is the unit vector of direction joining the charge q_1 to the charge q_2, directed from q_1 to q_2

;***$k=1/4\pi\varepsilon_0=8,98*10^{9}$ N m^2 C^2*** and we often use the approximate value: ***9.10^{9}*** or ***$\varepsilon_0=8,25*10^{-12}$ (SI)***, it represents the permittivity of a vacuum.

This expression is valid only for stationary charges located in a vacuum. It is considered as the Base foundation of all electrostatics. The electrostatic force possesses exactly the same vector properties as the force of gravity ***$F_g=G\,m_1\,m_2/r^2\,U$*** and ***follows the principle of action and reaction of classical mechanics.***



Repulsive force



Attractive force

➕ *Complément : What is the relationship between gravitational attraction and Coulomb repulsion between two electron?*

$F_e/F_g=e^2/(4\pi\varepsilon_0).\ 1/(Gm_e^{\,2})\approx 4.10^{42}$

The electrostatic force thus appears dominant compared to gravitational attraction. This implies that all celestial bodies are precisely

electrically neutral. Except that ***the gravitational force is always attractive***, but ***the electrostatic force can be attractive or repulsive.***

➕ *Complément : What is the Coulombic repulsion force between two charges of 1 C located 1 km apart?*

$F_e/g=1/(4\pi\varepsilon_0).\ 1/(10^{3})^{2}.\ 1/10\approx 10^{3}$ kg

This is a force equivalent to ***the weight exerted by a ton!***

II-1-3-1 Principle of superposition

Let us now consider a charge ***q*** placed at a point ***P*** and in the presence of other charges q_i located at points M_i, r_i is the distance from M_i to ***M***, and ***U*** is a unit vector along the direction from M_i to ***M***. The principle of superposition allows us to express the force F_e acting on the charge ***q*** in the following form: $F_e = \Sigma\, k\, q\, q_i / r^2$ (Figure II. 3).

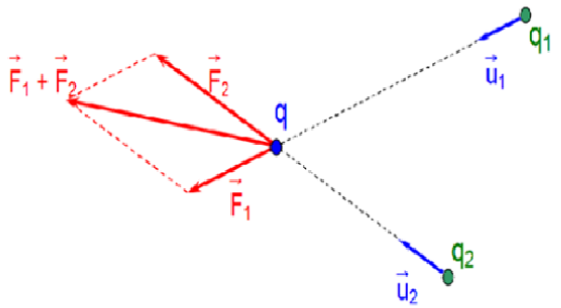


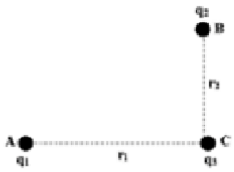
Figure II.3: Principle of superposition

Exemple

- Three charges q_1, q_2 and q_3 are arranged according to the figure,

- Calculate the net force applied to the charge q_3?

We give $q_1 = +1{,}5.\ 10^{-1}C$, $q_2 = -0{,}\ 5.\ 10^{-3}C$, $q_3 = +0{,}\ 2.\ 10^{-3}C$, $A\ C = 1{,}\ 2\ m$, $B\ C = 0{,}\ 5m$



Solution:

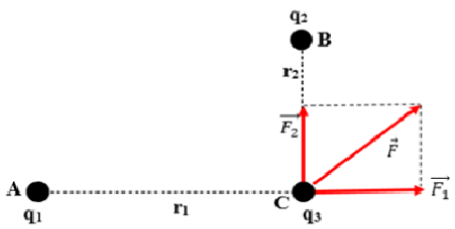
- q_1 and q_3 have the same sign, so the force F_1 is repulsive.

- q_2 and q_3 have an opposite sign, so the force F_2 is attractive.

$F_1 = k\, q_1\, q_3 / r_1^2 \rightarrow F_1 = 1,8.10^3\ N$

$F_2 = k\, q_2\, q_3 / r_2^2 \rightarrow F_2 = -3,6.10^3\ N$

Therefore: $F = F_1 + F_2 = 1,08.10^3\ N$



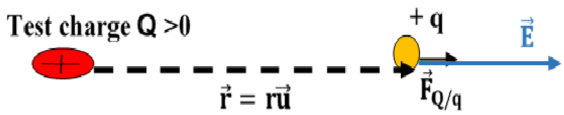
2.4. II-1-4 Electrostatic field

II-1-4-1 Electrostatic field created by a point charge:

By definition, we said that it exists an ***electric field*** at a given point in space where a test charge q_0 is located if this charge is subjected to a force: F_e such that: $E = F_e / q_0 = k\,Q / r^2$

In the international system of unit, the electric field $\vec{E}$ expressed by $V.m^{-1}$, E is parallel to F_e

The direction of E depends on the sign of q_0. If $q_0 > 0$, E and F_e have the same direction. If $q_0 < 0$, E and F_e have an opposite direction:



✎ *Remarque*

In every point in space there is a field created by the charge Q even if the test charge (q) does not exist.

II-1-4-2 Electrostatic field created by a set of point charges:

Let us consider n charges q_i located at points P_i, what would be the electric field produced by this set of charges at point M. As with forces, the principle of superposition is also valid for electric fields. The total field E_M is the vector sum of all contributions from each of the charges (see the Figure II.4). Therefore: $E_M = \Sigma\, k\, q_i / r^2$

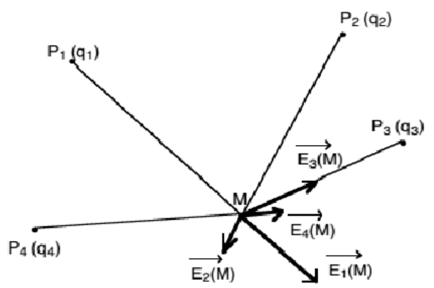


Figure II.4: Electrostatic field created by a set of point charges

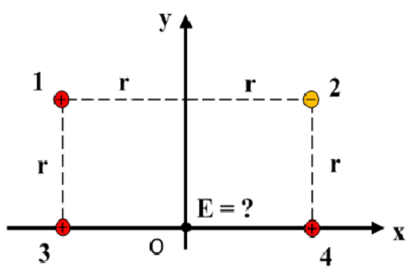
Exemple

Four point charges, all with the same magnitude ***|q|***, are placed around the origin ***O***.

q_1, q_3, q_4 have positive charges, while q_2 is negative.

- Find the direction of the electrostatic field ***E*** created by the ***4*** charges at point ***O***.

- An electron ***(-e)*** is placed at the origin; what is the direction of the force F_e acting on this charge?



Solution:

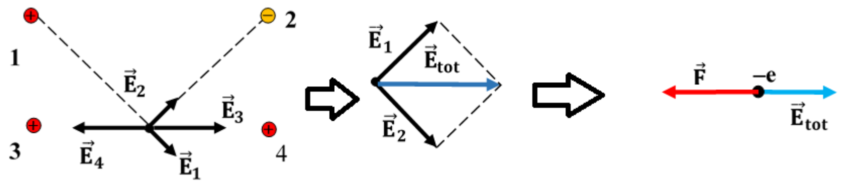
- The electrostatic field E_{tot} at the origin is the sum of the **4** vectors:

$E_{tot}=E_1+E_2+E_4=\Sigma E_i$

E_3 and E_4 are equal and in opposite directions: $E_4+E_3=0$

-The charge is negative, the direction of the force is opposite to the direction of E_{tot}:

$F=-e\,E_{tot}$



II-1-4-3 Electric field created by a continuous distribution of charges:

Let us consider a continuous distribution of charges within a certain volume, on a surface, or along a line:

**** Case of volume:***

The distribution is characterized at each point ***M*** within the volume by the specification of the volume charge density $\rho(P) = dq/dv$, where dq represents the electric charge contained in the element of volume dv surrounding point ***P***. In the case of a uniform charge distribution, dq is small enough to be considered as point-like, thus the electric field $(\vec{E})$ created at a point ***M*** by the charge dq is expressed as:

$dE=1/(4\pi\varepsilon_0)\,dq/r^2 \rightarrow dE=1/(4\pi\varepsilon_0)\,\rho\,dv/r^2$, We therefore write for the entire distribution:

$E=1/4\pi\varepsilon_0 \iiint \rho\,dv/r^2$

**** Case of Surface:***

For a surface distribution of charges characterized by the surface charge density $\sigma = dq/dS$ at each point on a surface , we will write similarly:

$E=1/4\pi\varepsilon_0 \iint \sigma\,ds/r^2$

* ***Case of Line***: For a linear charge distribution characterized at each point on a curve by the linear charge density $\lambda = dq/dl$:

$$E = 1/4\pi\varepsilon_0 \int \lambda \, dl / r^2$$

II-1-4-4 Field lines:

An electrostatic field line is a curve that is tangent at each point to the electrostatic field vector defined at that point. The set of field lines defines a map of the field:

- Two field lines never intersect at a point ***M*** unless the field is zero at ***M***.

- An electrostatic field line is not closed. It originates from a charge ***q*** and terminates

on a charge of opposite sign.

- To determine the direction of the field at a point ***M*** on a field line, a positive test charge

should be placed there, and the direction and sense of the electrostatic force it experiences

should be observed. These direction and sense are the same as those of the field. For a point charge, the field lines are half-lines intersecting at the location of the charge. ***If the charge is positive, the field is directed outward***, and it is said to be emanating; the field lines exhibit the same behavior. Conversely, ***for a negative charge, the field lines converge towards the charge,*** indicating that the field is directed toward the charge.

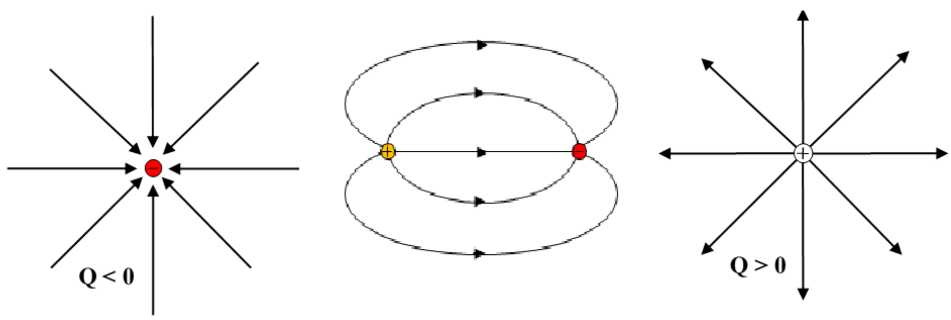


Figure II.5: Field lines direction.

2.5. II-2 Electrostatic potential

2.5.1. II-2 Electrostatic potential

Definition:

The Electrostatic potential is a scalar quantity denoted as ***V*** at a point, and it is equal to the electric potential energy (measured in Joules) per unit of charge (measured in Coulombs

$V=E\ p\ /\ q$ .The SI unit of potential is the Joule per Coulomb (***J / C***), a unit named Volt (***V***) in honor of the Italian scientist ***Alessandro Volta (1745-1827)***. « ***1 Volt= 1 Joule / C*** »

The work done to move the charge from position ***A*** to position ***B***:

$w_{A\,B}=\int F\ \ d\ r=q\int E\ d\ r\ \ ,\ w_{A\,B}=q\int k\ Q'/\ r^{2}\ d\ r=k\ q\ Q'\int d\ r\ /r^{2}=k\ q\ Q'\ (-1/\ r),\ w_{A\,B}=k\ q\ Q'\ (1/\ r_{B}\ -1/\ r_{A})$

This means that the work required to move the charge from point ***A*** to point ***B*** is independent of the path taken. When the circulation of the field along the curve does not depend on the path but only on the starting and ending points, in this case, we say that the field is ***conservative***. We pose $d\ V=-E\ d\ r=E\ p\ (B)-\ E\ p\ (A)\rightarrow E_{x}\ d\ x+E_{y}\ d\ y+E_{z}\ dz=(-d\ V/\ d\ x).\ d\ x-(-d\ V/\ d\ y).\ d\ y-(-d\ V/\ d\ z).\ dz$, it follows that: $E_{x}=(-d\ V/\ d\ x),\ E_{y}=(-d\ V/\ d\ y),\ E_{z}=(-d\ V/\ dz)\rightarrow grad\ V=(d\ V/\ d\ x)\ i+(d\ V/\ d\ y)\ j+(d\ V/\ dz)\ z$

Therefore, the previous relation can be condensed in the form:

$E=-\ grad\ V$

2.5.2. II-2-1 Electrostatic potential created by a point charge:

To obtain the potential ***V***, we first calculate the circulation of the field along any radius.

$d\ V=-E\ d\ r=(-q/\ 4\ \pi\ \varepsilon_{0})d\ r\ /\ r^{2}\ ,\ V=(-q\ /\ 4\ \pi\ \varepsilon_{0})\int d\ r\ /\ r^{2}=(q\ /\ 4\ \pi\ \varepsilon_{0})\ 1/r+C^{t\,e}$

Assuming that ***V=0*** when ***r*** tends towards **∞** we will have the $C^{t\,e}=0$ ***volts***. The Potential

V is positive or negative depending on the sign of charge, which produces it. ***The potential is constant on spheres of radius r*** whose center is the charge ***q***. These spheres are said to constitute ***equipotential surfaces***.

2.5.3. II-2-2 Electrostatic potential created by a set of point charges:

If there are multiple charges $q_1,\ q_2,\ q_3$, the potential at a point ***P*** is the sum of their individual potentials:

$V=q_{1}/\ (4\ \pi\ \varepsilon_{0}\ r_{1})+q_{2}/(4\ \pi\ \varepsilon_{0}\ r_{2})+.........+q_{i}/\ (\ 4\ \pi\ \varepsilon_{0}\ r_{i})$

Where r_i is the distance between q_i and point ***p***. The charge q_i can be positive or negative.

Exemple

calculate the electric field ***E*** produced by a point charge from the given value of the

potential ***V***: $V=q\ /4\ \pi\ \varepsilon_{0}\ r$

Solution:

We have: $-E = dV / dr = d/dr\,(q/4\pi\varepsilon_0 r) = q/(4\pi\varepsilon_0)\, d/dr\,(1/r)$, $E = q/(4\pi\varepsilon_0 r^2)$

2.5.4. II-2-3 Electrostatic potential created by a continuous distribution of charges:

In this case, we must carry out an integration after having chosen an elementary charge

corresponding, with the same process as that of the electric field for such a case:

$dV = (1/4\pi\varepsilon_0)\, dq / r$, $V(P) = 1/(4\pi\varepsilon_0) \int dq / r$. In the general case it is preferable to calculate the potential first, then deduce the electric field by derivation. It is assumed that the charge distribution is uniform throughout our study:

* ***Case of volume:*** $V(P) = \iiint \rho\, dv / (4\pi\varepsilon_0 r)$

* ***Case of Surface:*** $V(P) = \iint \sigma\, ds / (4\pi\varepsilon_0 r)$

**** Case of Line:*** $V(P) = \int \lambda\, dl / (4\pi\varepsilon_0 r)$

2.5.5. II-2-4 Electrostatic energy:

The electrostatic energy W of a system of charges, initially assumed to be distant from each other, corresponds to the work that must be done to bring these charges to their final positions. For a charge ***q*** moving from ***A*** to ***B*** in the field ***E***, the work of the electrostatic force is: $W_{AB} = q\,(V_B - V_A) = q\,V$

III-2-4-1 Electrostatic energy created by a continuous distribution of charges:

**** Volume distribution :*** $W = 1/2 \iiint \rho\, V\, dv$

**** Surface distribution :*** $W = 1/2 \iint \sigma\, V\, ds$

8 Linear distribution : $W = 1/2 \int \lambda\, V\, dl$

The term ½ comes from the fact that in the interaction between q_i and q_j is counted twice.

2.5.6. Exercice : Choose the right answers:

1- Electrostatics is the branch of physics that studies:

- [ ] The charges stability

- [ ] The charges instability

2.5.7. Exercice

2- The Electric charge can be :

- [ ] Positive

- [ ] Negative

2.5.8. Exercice

3- In the international system of unit, the electric field **E** expressed by:

- ☐ $V.\ m^{-1}$
- ☐ $C.\ m$

2.5.9. Exercice

4- The Electrostatic force can be :

- ☐ Attractive
- ☐ Repulsive

2.5.10. Exercice

5- The electric field created by a continuous distribution of charges along a line is :

- ☐ $\rho = dq / dv$
- ☐ $\lambda = dq / dl$

2.5.11. Exercice

6- The relationship between the electric field $\mathbf{E}$ and the electric potential V is:

- ☐ $E = -\ grad\ \ V$
- ☐ $V = \int E\ dr$

2.6. TD N 2

Exercise 1:

Complete the following table:

Physical quantity	Symbol	Expression	Unit
The charge			
The voltage			
The Capacity			
The resistance			
The current			

Exercice 2 :

- Consider a charge $\mathbf{q_3}$in the presence of charges $\mathbf{q_1}$and $\mathbf{q_2}$as shown in the figure.
- Calculate the resultant force acting on $\mathbf{q_3}$ with $\mathbf{q_1= -2.5\ 10^{-3}\ C}$, $\mathbf{q_2= 1.5\ 10^{-3}C}$,

$\mathbf{q3= 0.8\ 10^{-3}}$ C, $\mathbf{r_1= AC= 1.2\ m}$, and $\mathbf{r_2= BC= 0.8\ m.}$

Exercise 3:

Consider three capacitors with capacitances of **3, 6,** and **12 µF**.

- Determine their equivalent capacitance when they are connected

a. In parallel

b. In series

c. mixed connection : 2 parallel and 1 in series with them.

- In each case, plot the possible circuit.

3. Final Exam

[cf. final exam]

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