

Correction de l'exercice 1

$$\vec{u} + \vec{v} = \begin{pmatrix} 2 + (-4) \\ -1 + 2 \\ 3 + 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}.$$

$$\vec{u} - \vec{v} = \begin{pmatrix} 2 - (-4) \\ -1 - 2 \\ 3 - 1 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix}.$$

$$3\vec{w} = \begin{pmatrix} 3 \\ -6 \\ 12 \end{pmatrix}, \quad -\frac{1}{2}\vec{w} = \begin{pmatrix} -\frac{1}{2} \\ 1 \\ -2 \end{pmatrix}.$$

Correction de l'exercice 2

$$\vec{a} \cdot \vec{b} = 2(1) + (-1)(4) + 3(-2) = 2 - 4 - 6 = -8.$$

$$\|\vec{a}\| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{4 + 1 + 9} = \sqrt{14}, \quad \|\vec{b}\| = \sqrt{1^2 + 4^2 + (-2)^2} = \sqrt{21}.$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} = \frac{-8}{\sqrt{14}\sqrt{21}} = \frac{-8}{\sqrt{294}}.$$

$$\text{Donc } \theta = \arccos\left(\frac{-8}{\sqrt{294}}\right) \approx 118,9^\circ.$$

$$\vec{u} \times \vec{v} = \begin{pmatrix} u_2v_3 - u_3v_2 \\ u_3v_1 - u_1v_3 \\ u_1v_2 - u_2v_1 \end{pmatrix}.$$

$$\vec{u} \times \vec{v} = \begin{pmatrix} 2(1) - 3(-1) \\ 3(2) - 1(1) \\ 1(-1) - 2(2) \end{pmatrix} = \begin{pmatrix} 2 + 3 \\ 6 - 1 \\ -1 - 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 5 \\ -5 \end{pmatrix}.$$

Correction de l'exercice 3

$$\|\vec{a}\| = \sqrt{4^2 + (-2)^2 + 4^2} = \sqrt{16 + 4 + 16} = \sqrt{36} = 6.$$

$$\vec{u}_a = \frac{1}{\|\vec{a}\|} \vec{a} = \frac{1}{6} \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{pmatrix}.$$

Correction de l'exercice 4

$$\vec{p} \cdot \vec{q} = 1(2) + 2(-1) + (-1)(0) = 2 - 2 + 0 = 0.$$

$$\rightarrow \vec{p} \perp \vec{q}.$$

Correction de l'exercice 5

$$\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v}.$$

$$\vec{u} \cdot \vec{v} = 3(1) + 1(1) + 2(1) = 6, \quad \|\vec{v}\|^2 = 1^2 + 1^2 + 1^2 = 3.$$

$$\Rightarrow \text{proj}_{\vec{v}} \vec{u} = \frac{6}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}.$$