Correction de l'exercice 1

Point cartésien $P(1,-\sqrt{3},2)$.

Cylindriques (r,θ,z)

$r=\sqrt{1^2+(-\sqrt{3})^2}=\sqrt{1+3}=2.$

$\theta=\operatorname{atan2}(-\sqrt{3},1)=-\dfrac{\pi}{3}$ (ou $5\pi/3$).

$z=2.$

Donc $\boxed{(r,\theta,z)=(2,\ -\frac{\pi}{3},\ 2)}$.

Sphériques (ρ,θ,φ)

$\rho=\sqrt{1+3+4}=\sqrt{8}=2\sqrt{2}.$

θ identique : $-\frac{\pi}{3}$.

$\varphi=\arccos\frac{z}{\rho}=\arccos\frac{2}{2\sqrt{2}}=\arccos\frac{1}{\sqrt{2}}=\frac{\pi}{4}.$

Donc $\boxed{(\rho,\theta,\varphi)=\left(2\sqrt{2},\ -\frac{\pi}{3},\ \frac{\pi}{4}\right)}$.

Correction de l'exercice 2

1. $A(3,3,2)$ → cylindriques :
   $r=\sqrt{3^2+3^2}=3\sqrt{2},\ \theta=\arctan(1)=\frac{\pi}{4},\ z=2.$
   $\boxed{(r,\theta,z)=(3\sqrt{2},\ \frac{\pi}{4},\ 2)}$.
2. $B(-2,2,-1)$ → cylindriques :
   $r=\sqrt{(-2)^2+2^2}=2\sqrt{2},\ \theta=\operatorname{atan2}(2,-2)=\frac{3\pi}{4},\ z=-1.$
   $\boxed{(r,\theta,z)=(2\sqrt{2},\ \frac{3\pi}{4},\ -1)}$.
3. $A(2,2,1)$ → sphériques :
   $\rho=\sqrt{2^2+2^2+1^2}=3,\ \theta=\frac{\pi}{4},\ \varphi=\arccos\frac{1}{3}.$
   $\boxed{(\rho,\theta,\varphi)=\left(3,\ \frac{\pi}{4},\ \arccos(1/3)\right)}$.
4. $B(-1,-1,2)$ → sphériques :
   $\rho=\sqrt{1+1+4}=\sqrt{6},\ \theta=\operatorname{atan2}(-1,-1)=-\frac{3\pi}{4}$ (ou $5\pi/4$),
   $\varphi=\arccos\frac{2}{\sqrt{6}}.$
   $\boxed{(\rho,\theta,\varphi)=\left(\sqrt{6},\ -\frac{3\pi}{4},\ \arccos(2/\sqrt{6})\right)}$.

Correction de l'exercice 3

(on donne des points en cylindriques → passer en cartésiennes)

1. $A(2, \frac{\pi}{3}, 5)$:
   $x = 2\cos\frac{\pi}{3} = 2 \cdot \frac{1}{2} = 1,\ y = 2\sin\frac{\pi}{3} = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3},\ z = 5.$
   $\boxed{A_{\text{cart}} = (1,\ \sqrt{3},\ 5)}.$
2. $B(3, \pi, -2)$:
   $x = 3\cos\pi = 3 \cdot (-1) = -3,\ y = 3\sin\pi = 0,\ z = -2.$
   $\boxed{B_{\text{cart}} = (-3,\ 0,\ -2)}.$

Correction de l'exercice 4

(points sphériques (ρ, θ, φ) → cartésiennes)

1. $A(5, \frac{\pi}{3}, \frac{\pi}{6})$:
   $\sin\varphi = \sin\frac{\pi}{6} = \frac{1}{2},\ \cos\varphi = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}.$
   $x = 5 \cdot \frac{1}{2} \cdot \cos\frac{\pi}{3} = 5 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{4}.$
   $y = 5 \cdot \frac{1}{2} \cdot \sin\frac{\pi}{3} = 5 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{4}.$
   $z = 5 \cdot \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2}.$
   $\boxed{A_{\text{cart}} = \left(\frac{5}{4},\ \frac{5\sqrt{3}}{4},\ \frac{5\sqrt{3}}{2}\right)}.$
2. $B(2, \pi, \frac{\pi}{2})$: ici $\sin\varphi = 1,\ \cos\varphi = 0.$
   $x = 2\cos\pi = -2,\ y = 2\sin\pi = 0,\ z = 0.$
   $\boxed{B_{\text{cart}} = (-2,\ 0,\ 0)}.$

Correction de **l'exercice** 5

1. $A(3, \frac{\pi}{4}, 4)$ — (cylindriques (r, θ, z) → sphériques (ρ, θ, φ)) :
   $\rho = \sqrt{3^2 + 4^2} = 5,\ \theta = \frac{\pi}{4},\ \varphi = \arccos\frac{z}{\rho} = \arccos\frac{4}{5}.$
   $\boxed{(5,\ \frac{\pi}{4},\ \arccos(4/5))}.$
2. $B(2, \pi, 0)$:
   $\rho = \sqrt{2^2 + 0^2} = 2,\ \theta = \pi,\ \varphi = \arccos\frac{0}{2} = \frac{\pi}{2}.$
   $\boxed{(2,\ \pi,\ \frac{\pi}{2})}.$

Correction de **l'exercice** 6

(points sphériques → cylindriques)

1. $A(4, \frac{\pi}{3}, \frac{\pi}{6})$:
   $r = \rho \sin\varphi = 4 \sin\frac{\pi}{6} = 4 \cdot \frac{1}{2} = 2,\ \theta = \frac{\pi}{3},\ z = \rho\cos\varphi = 4\cos\frac{\pi}{6} = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}.$
   $\boxed{(r, \theta, z) = (2,\ \frac{\pi}{3},\ 2\sqrt{3})}.$
2. $B(5, \pi, \frac{\pi}{2})$:
   $r = 5\sin\frac{\pi}{2} = 5,\ \theta = \pi,\ z = 5\cos\frac{\pi}{2} = 0.$
   $\boxed{(r, \theta, z) = (5,\ \pi,\ 0)}.$