

Tutorial N° 2

Exercise 1. *Let*

$$\vec{u} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}.$$

Compute: $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, $3\vec{w}$, and $-\frac{1}{2}\vec{w}$.

Exercise 2. *Let*

$$\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}, \quad \vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}.$$

Compute: $\vec{a} \cdot \vec{b}$ and the angle θ between $\vec{a}$ and $\vec{b}$. Compute $\vec{u} \times \vec{v}$.

Exercise 3. *Let*

$$\vec{a} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}.$$

- 1. Compute its norm $\|\vec{a}\|$.*
- 2. Find the unit vector $\vec{u}_a$ in the same direction.*

Exercise 4. *Let*

$$\vec{p} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \quad \vec{q} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}.$$

Check whether $\vec{p}$ and $\vec{q}$ are orthogonal.

Exercise 5. *Let*

$$\vec{u} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Calculate the projection of $\vec{u}$ onto $\vec{v}$.

Solution to Exercise 1

Let

$$\vec{u} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}, \quad \vec{w} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}.$$

$$\vec{u} + \vec{v} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{u} - \vec{v} = \begin{pmatrix} 1 \\ 1 \\ -4 \end{pmatrix},$$

$$3\vec{w} = \begin{pmatrix} -6 \\ 3 \\ 12 \end{pmatrix}, \quad -\frac{1}{2}\vec{w} = \begin{pmatrix} 1 \\ -\frac{1}{2} \\ -2 \end{pmatrix}.$$

Solution to Exercise 2

Let

$$\vec{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}, \quad \vec{u} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \vec{v} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}.$$

$$\vec{a} \cdot \vec{b} = 2(1) + (-1)(4) + 3(-2) = 2 - 4 - 6 = -8.$$

$$\|\vec{a}\| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{4 + 1 + 9} = \sqrt{14},$$

$$\|\vec{b}\| = \sqrt{1^2 + 4^2 + (-2)^2} = \sqrt{1 + 16 + 4} = \sqrt{21}.$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} = \frac{-8}{\sqrt{14}\sqrt{21}} = -\frac{8}{\sqrt{294}}.$$

Hence

$$\theta = \arccos \left(-\frac{8}{\sqrt{294}} \right) \approx 118.9^\circ.$$

$$\vec{u} \times \vec{v} = \begin{pmatrix} u_2v_3 - u_3v_2 \\ u_3v_1 - u_1v_3 \\ u_1v_2 - u_2v_1 \end{pmatrix} = \begin{pmatrix} 2(6) - 3(5) \\ 3(4) - 1(6) \\ 1(5) - 2(4) \end{pmatrix} = \begin{pmatrix} 12 - 15 \\ 12 - 6 \\ 5 - 8 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \\ -3 \end{pmatrix}.$$

Solution to Exercise 3

$$\|\vec{a}\| = \sqrt{4^2 + (-2)^2 + 4^2} = \sqrt{16 + 4 + 16} = \sqrt{36} = 6.$$

$$\vec{u}_a = \frac{1}{\|\vec{a}\|} \vec{a} = \frac{1}{6} \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{pmatrix}.$$

Solution to Exercise 4

$$\vec{p} \cdot \vec{q} = 1(2) + 2(-1) + (-1)(0) = 2 - 2 + 0 = 0.$$

Hence $\vec{p} \perp \vec{q}$.

Solution to Exercise 5

$$\vec{u} \cdot \vec{v} = 3(1) + 1(1) + 2(1) = 6,$$

$$\|\vec{v}\|^2 = 1^2 + 1^2 + 1^2 = 3.$$

Thus

$$\text{proj}_{\vec{v}} \vec{u} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v} = \frac{6}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 2\vec{v}$$